Properties of Real Numbers

Objective To identify and use properties of real numbers

Tell whether each pair of expressions is equal by completing each statement with = or ≠. Explain your answers.

$34 + 12 \stackrel{=}{?} 12 + 34$ $18 \div \frac{1}{18} \stackrel{\neq}{?} 1$

$100 - 1 \stackrel{\neq}{?} 1 - 100$ $45 - 1 \stackrel{\neq}{?} 45$

$0 + 180 \stackrel{?}{=} 180$ $6 \times \frac{1}{6} \stackrel{\neq}{?} 1$
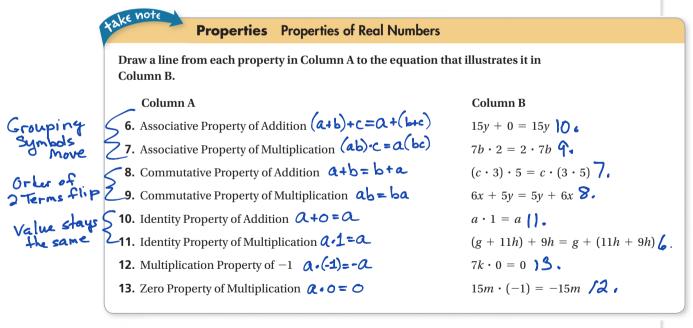
Remember that ≠ means "is not equal to."

Relationships that are always true for real numbers are called *properties*, which are rules used to rewrite and compare expressions.

Two algebraic expressions are **_equivalent expressions_** *if they have the same value for all values of the variable(s).* The following properties show expressions that are equivalent for all real numbers.

take note

Properties   Properties of Real Numbers

Draw a line from each property in Column A to the equation that illustrates it in Column B.

Grouping Symbols Move

Order of 2 Terms flip

Value stays the same

Column A

6. Associative Property of Addition $(a+b)+c = a+(b+c)$
7. Associative Property of Multiplication $(ab)\cdot c = a(bc)$
8. Commutative Property of Addition $a+b = b+a$
9. Commutative Property of Multiplication $ab = ba$
10. Identity Property of Addition $a+0 = a$
11. Identity Property of Multiplication $a\cdot 1 = a$
12. Multiplication Property of -1 $a\cdot(-1) = -a$
13. Zero Property of Multiplication $a\cdot 0 = 0$

Column B

$15y + 0 = 15y$ 10.

$7b \cdot 2 = 2 \cdot 7b$ 9.

$(c \cdot 3) \cdot 5 = c \cdot (3 \cdot 5)$ 7.

$6x + 5y = 5y + 6x$ 8.

$a \cdot 1 = a$ 11.

$(g + 11h) + 9h = g + (11h + 9h)$ 6.

$7k \cdot 0 = 0$ 13.

$15m \cdot (-1) = -15m$ 12.

PROBLEM 1: IDENTIFYING PROPERTIES

What property is illustrated by each statement?

a). $42 \cdot 0 = 0$

Zero Property of Mult.

b). $(y + 25) + 28 = y + (25 + 28)$

Assoc. Prop. of Add.

c). $10x + 0 = 10x$

Identity Prop. of Add.

d). $4x \cdot 1 = 4x$

Identity Prop. of Mult.

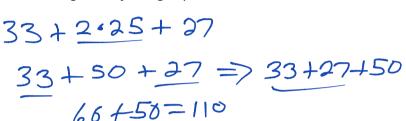e). $x + (\sqrt{y} + z) = x + (z + \sqrt{y})$

Comm. Prop. of Add.

PROBLEM 2: USING PROPERTIES FOR MENTAL CALCULATIONS

a) A movie ticket costs $7.75. A drink costs $2.40. Popcorn costs $1.25. What is the total cost for a ticket, a drink, and popcorn? Use mental math.

$$7.75 + 2.40 + 1.25 \Rightarrow 7.75 + 1.25 + 2.40$$

$$9 + 2.40$$
$$11.40$$

b) The sign at the right shows the costs for a deep=sea fishing trip. How much will the total cost be for 1 adult, 2 children, and 1 senior citizen to go on a fishing trip? Use mental math.

DEEP-SEA FISHING
Adults $33
Children (12 & under)..........$25
Seniors (65 & up)................$27

$$33 + 2 \cdot 25 + 27$$

$$33 + 50 + 27 \Rightarrow 33 + 27 + 50$$
$$66 + 50 = 110$$

PROBLEM 3: WRITING EQUIVALENT EXPRESSIONS

Simplify each expression.

a) $5(3n)$ — Justification

$(5 \cdot 3)n$ Assoc. Prop. of Mult.

$15n$

b). $(4 + 7b) + 8$

$4 + (7b + 8)$ Assoc Prop. of Add.
$4 + (8 + 7b)$ Comm. Prop. of Add.
$(4 + 8) + 7b$ Assoc. Prop of Add
$12 + 7b$ Simplify

c). $\frac{6xy}{y}$

$\frac{6 \cdot x \cdot y}{1 \cdot 1 \cdot y}$ Identity Prop of Mult

$\frac{6}{1} \cdot \frac{x}{1} \cdot \frac{y}{y}$ $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

$6 \cdot x \cdot 1$ $\frac{y}{y} = a$ and $\frac{a}{a} = 1$

$6x$ Identity Prop. of Mult.

d) $2.1(4.5x)$

$(2.1 \cdot 4.5)x$

$9.45x$

e). $6 + (4h + 3)$

$6 + (3 + 4h)$
$(6 + 3) + 4h$
$9 + 4h$

f). $\frac{2.8mn}{3.12mn}$

$\frac{8}{12} \cdot \frac{m}{m} \cdot \frac{1}{n}$

$\frac{2}{3} \cdot 1 \cdot \frac{1}{n}$

$\frac{2}{3n}$

In problem 3, reasoning and properties were used to show that two expressions are equivalent. This is an example of *deductive reasoning*. ***Deductive reasoning*** *is the process of reasoning logically from given facts to a conclusion.*

To show that a statement is NOT true, find an example for which it is not true. *An example showing that a statement is false is a **counterexample.*** You need only one counterexample to prove that a statement is false.

Give a counterexample that proves each statement is false.

a) If you live near an ocean, you live near the Atlantic Ocean.

False; You live near the Pacific Ocean.

b) If you live in North America, you live in the United States.
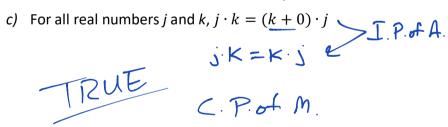
False; Canada or Mexico

c) If you live in Miami, you live in Florida.

False; Miami, Ohio

PROBLEM 4: USING DEDUCTIVE REASONING AND COUNTEREXAMPLES

Is the statement true or false? If it is false, give a counterexample.

a) For all real numbers a and b, $a \cdot b = b + a$

$a = 9$
$b = 10$

$9 \cdot 10 = 10 + 9$
$90 \neq 19$

FALSE

$a = 10$
$b = 4$

$10 \cdot 4 = 4 + 10$
$40 \neq 14$

$a = -20$
$b = 5$

$-20 \cdot 5 = 5 + (-20)$
$-100 \neq -15$

b) For all real numbers a, b, and c, $(a + b) + c = b + (a + c)$

C.P. of A $(b+a) + c = b + (a+c)$

A.P. of A $b + (a+c) = b + (a+c)$

TRUE

c) For all real numbers j and k, $j \cdot k = (k + 0) \cdot j$

$j \cdot k = k \cdot j$ ⟵ I.P. of A.

TRUE   C.P. of M.

d) For all real numbers m and n, $m(n + 1) = mn + 1$

$m = 2$
$n = 4$

$2(4+1) = 2(4) + 1$
$2(5) = 8+1$
$10 \neq 9$

FALSE

e) Is the statement in part a) false for *every* pair of real numbers a and b? Explain.

$1.5 \cdot 3 = 1.5 + 3$
$4.5 = 4.5$

$a \cdot b = b + a$
$-a \quad -a$
$ab - a = b$
$\dfrac{a(b-1)}{b-1} = \dfrac{b}{b-1}$

$b = -1$

$a = \dfrac{-1}{-1-1} \cdot \dfrac{-1}{-2} = \dfrac{1}{2}$

$\frac{1}{2}(-1) = -1 + \frac{1}{2}$
$-\frac{1}{2} = -\frac{1}{2}$

$$a = \frac{b}{b-1}$$

✓

Lesson Check • Do you UNDERSTAND?

Justify each step to show that 3 · (10 · 12) = 360.

29. The left side of the expression is simplified below. Write a reason for each step.

3 · (10 · 12) = 3 · (12 · 10)   C. P. of M.

= (3 · 12) · 10   A. P. of M.

= 36 · 10   multiply

= 360   multiply

Math Success

Check off the vocabulary words that you understand.

☐ Commutative Properties   ☐ Associative Properties   ☐ Identity Properties

☐ equivalent expressions   ☐ deductive reasoning   ☐ counterexample

Rate how well you can *use the properties of addition and multiplication*.

| Need to review | 0 | 2 | 4 | 6 | 8 | 10 | Now I get it! |